Probability of Weekly Aircraft Sales

Posted on March 25, 2019

The aircraft-sales problem provides fifty weeks of historical observations and asks for the probability of selling zero, one, two, or three planes in a week. The original calculations are correct: zero aircraft were sold in forty weeks, one in eight weeks, two in one week, and three in one week. The original explanation misidentifies the method as subjective probability. Because the estimates come directly from observed frequencies, the best approach is empirical or relative-frequency probability. Subjective probability would rely primarily on informed judgment when adequate repeat data are unavailable. This distinction matters because the selected method determines how the result should be interpreted and how confidently it can be used for forecasting.

Organizing the Historical Sales Data

Let the random variable X represent the number of aircraft sold during one week. The observed outcomes are 0, 1, 2, and 3. There are fifty observations in total. Before calculating probability, the frequencies should be checked to confirm that they sum to fifty:

40 + 8 + 1 + 1 = 50 weeks.

The relative frequency of an outcome is its number of occurrences divided by the total number of observations. The estimated probability distribution is therefore based on the sales office’s experience during the observed period. It is not a guarantee that the next fifty weeks will repeat the same counts.

Probability of Selling Zero Aircraft

Zero aircraft were sold in forty of the fifty weeks. The empirical probability is:

$P(X = 0) = 40/50 = 0.80$.

This means that 80 percent of the observed weeks had no aircraft sale. It does not mean that there is an independently proven 80 percent chance in every future week under all conditions. The estimate assumes that the historical period is sufficiently representative of the future environment. Changes in demand, staffing, price, inventory, economic conditions, or marketing can alter the distribution.

Probability of Selling One Aircraft

One aircraft was sold in eight weeks. Therefore:

$P(X = 1) = 8/50 = 0.16$.

The historical relative frequency is 16 percent. The fact that one sale occurred more often than two or three sales makes it the most common positive-sales outcome, but weeks with no sales remain much more common.

Probability of Selling Two Aircraft

Two aircraft were sold in one of the fifty weeks:

$P(X = 2) = 1/50 = 0.02$.

The observed probability is 2 percent. Because it is based on only one occurrence, the estimate is highly uncertain. One additional two-aircraft week would double the count and noticeably change the proportion. Rare-outcome estimates require more data before they can be considered stable.

Probability of Selling Three Aircraft

Three aircraft were sold in one week:

$P(X = 3) = 1/50 = 0.02$.

This is also an empirical probability of 2 percent. There were no reported weeks with more than three sales. The absence of such an outcome in fifty observations does not prove it is impossible; it only means the sample did not include it.

Checking the Probability Distribution

A valid discrete probability distribution has probabilities between zero and one that sum to one. Here:

$0.80 + 0.16 + 0.02 + 0.02 = 1.00$.

The distribution therefore accounts for every observed weekly outcome. It can be used to calculate additional descriptive quantities, provided the historical-frequency assumption is stated.

Probability of at Least One Sale

The probability of selling at least one aircraft includes outcomes of one, two, or three. These can be added:

$P(X \geq 1) = 0.16 + 0.02 + 0.02 = 0.20$.

The same result can be obtained through the complement:

$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.80 = 0.20$.

Thus, a sale occurred in 20 percent of observed weeks. This may be useful when managers need to estimate the likelihood that a week generates any aircraft sale rather than the exact number.

Expected Weekly Aircraft Sales

The expected value is the long-run average implied by the probability distribution. It is calculated by multiplying each outcome by its probability and adding the products: (Ross, 2019; Triola, 2018)

$E(X) = (0 \times 0.80) + (1 \times 0.16) + (2 \times 0.02) + (3 \times 0.02)$.

$E(X) = 0 + 0.16 + 0.04 + 0.06 = 0.26$ aircraft per week.

The same result follows from total sales divided by total weeks. Historical sales equal $0(40) + 1(8) + 2(1) + 3(1) = 13$ aircraft; $13/50 = 0.26$. The expected value does not imply that 0.26 of a physical aircraft will be sold next week. It is an average over many weeks.

Expected Sales Over a Longer Period

If the weekly distribution remains approximately stable, expected sales over ten weeks would be: (Garvey et al., 2016)

$10 \times 0.26 = 2.6$ aircraft.

Over fifty future weeks, the expectation would be 13 aircraft. These are planning averages rather than exact forecasts. Actual sales may be clustered, seasonal, or influenced by a few large customers. Multiplying the average assumes each future week has a similar expected value.

Variation in Weekly Sales

The expected value alone does not show uncertainty. The second moment is:

$E(X^2) = (0^2 \times 0.80) + (1^2 \times 0.16) + (2^2 \times 0.02) + (3^2 \times 0.02) = 0.42$.

The variance is:

$\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = 0.42 - 0.26^2 = 0.3524$.

The standard deviation is approximately:

$\sqrt{0.3524} \approx 0.594$ aircraft.

This calculation summarizes dispersion in the empirical model. The standard deviation exceeds the mean because most weeks have zero sales while a small number have multiple sales.

Which Probability Approach Is the Best Fit?

Three common introductory approaches are classical probability, empirical probability, and subjective probability. The aircraft problem is best addressed through empirical probability because actual historical frequencies are supplied. The value 40/50 is observed rather than derived from equally likely theoretical outcomes or personal belief.

Classical Probability

Classical probability applies when outcomes can reasonably be treated as equally likely by design or symmetry. For a fair six-sided die, each face has probability 1/6. For an ideal fair coin, heads and tails each have probability 1/2. Aircraft sales are not equally likely categories. There is no physical symmetry making zero, one, two, and three sales equally probable. Classical probability is therefore not the best primary method.

Empirical or Relative-Frequency Probability

Empirical probability estimates the chance of an outcome from the proportion observed in repeat data. It is appropriate here because fifty weekly records are available. The method is transparent and can be updated when additional weeks are observed. Its limitation is dependence on the representativeness and quality of the historical sample.

Subjective Probability

Subjective probability represents a reasoned degree of belief based on experience, expert judgment, market information, and other evidence. It is useful when events are unique, data are scarce, or conditions have changed so much that historical frequencies are poor guides. A sales manager might use subjective judgment for a new aircraft model with no history. In the current problem, subjective information could supplement the empirical estimate, but it should not replace or be confused with the frequency calculation.

Why the Original Classification Was Incorrect

The original answer says the subjective method is best because information about previous years is provided. That description actually defines the empirical approach. Subjective probability would arise if a manager said, after reviewing market conditions and customer discussions, “I believe there is a 30 percent chance of a sale next week,” even though that value was not the direct proportion of historical weeks.

Correct terminology strengthens the analysis because it tells readers where the number came from. An empirical probability can be checked by examining the data. A subjective probability must be evaluated through the quality and calibration of the expert judgment.

Sample Size and Uncertainty

Fifty weeks provide useful evidence but remain a limited sample, especially for rare outcomes. The estimates of 2 percent for two and three sales each are based on one week. A longer time series would reveal whether these categories recur consistently. Confidence intervals or Bayesian updating could express uncertainty rather than presenting each proportion as an exact population probability.

Historical records should also be checked for completeness. A week may be classified by contract signing, delivery, payment, or order acceptance. The definition must remain consistent. Missing or duplicate records can distort probabilities.

Stationarity and Changing Business Conditions

Relative-frequency forecasting assumes that the process generating future sales resembles the historical process. Aircraft markets can change because of interest rates, fuel prices, regulation, model availability, economic cycles, financing, fleet replacement, competition, or major contracts. A distribution measured during a recession may understate sales during expansion.

Managers should segment data where justified. Sales may differ by quarter, salesperson, aircraft type, customer region, or promotional period. Segmenting too aggressively produces very small samples, so the additional detail must be balanced with reliability.

Independence Between Weeks

The simple model treats each week as an observation from one distribution, but sales may not be independent. Negotiations can last months, and a sale completed this week may reduce the chance of another from the same customer soon. Multiple sales may occur together under a fleet agreement. Sales staff capacity and inventory constraints can also connect weeks.

If dependence or seasonality is substantial, a time-series, count-data, or pipeline model may forecast better than one unconditional frequency table. The empirical distribution remains a useful descriptive starting point.

Would GPS Installation Change the Probability Approach?

The original answer says GPS would not change the method because a sold plane cannot be sold again. That reasoning misunderstands the question. Installing GPS on some aircraft does not inherently affect whether relative-frequency probability is appropriate. The method depends on the type of data and question, not on whether the individual aircraft can be resold. If the office continues counting weekly aircraft sales, the historical relative-frequency method remains the same.

GPS becomes relevant only if it introduces another variable, such as whether each sold aircraft included a particular GPS package. The dataset could then record both the number sold and GPS status. Managers might estimate the joint probability of selling two aircraft with at least one GPS-equipped, the conditional probability that a sold aircraft includes GPS, or differences in demand between equipped and unequipped models.

Conditional and Joint Probability With GPS

Suppose records show how many sold aircraft were equipped with GPS. The conditional probability could be written:

P(GPS | aircraft sold) = number of sold aircraft with GPS / total aircraft sold.

A joint probability could estimate the chance of a week with at least one sale and at least one GPS-equipped aircraft. These calculations require additional data. GPS tracking of the physical aircraft after sale does not by itself improve the accuracy of sales records unless the system is designed to verify transaction or delivery information.

Privacy and Operational Limits of GPS Data

If GPS data refer to aircraft location after sale, the company must consider customer agreement, privacy, cybersecurity, ownership, and regulatory requirements. Location monitoring should not be added merely because the technology exists. It may be appropriate for fleet operations or service contracts but is separate from estimating weekly sales probability.

Using the Distribution for Management

The empirical distribution can inform staffing, cash-flow scenarios, and sales expectations. Because 80 percent of weeks had no sale, weekly revenue is likely to be uneven. Management should avoid evaluating a salesperson solely on one week. Pipeline stage, customer contacts, proposals, and longer-period results may provide a fairer view.

The distribution can also support simulation. A model can draw weekly outcomes according to the estimated probabilities and examine possible annual totals. Such simulations should incorporate parameter uncertainty and changing market conditions rather than treating 0.80, 0.16, 0.02, and 0.02 as timeless laws.

Conclusion

The empirical probabilities are 0.80 for zero aircraft, 0.16 for one, 0.02 for two, and 0.02 for three. They form a valid distribution summing to one. The probability of at least one sale is 0.20, and the expected number of sales is 0.26 per week. The correct method is relative-frequency probability because the estimates are calculated from fifty historical weeks. Classical probability is unsuitable because the sales outcomes are not equally likely by design, while subjective probability would rely on informed judgment rather than direct frequencies. Adding GPS does not automatically change the approach; it creates a new variable only if GPS status or location is part of the business question. The model is useful when its assumptions, limited sample, rare outcomes, and changing market conditions are stated clearly.

References

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