

Interdependent Parallel-Server Queueing with Vacations and Varying Arrivals

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Abstract

The M/M/1 interdependent queueing model with controlled arrival rates has been expanded into multi-server queueing model. When $c=1,2$, we take into account an interdependent queueing model with finite capacity and regulated arrival rates. When $c=1$, it denotes that there is only one service station, and when $c=2$, it denotes the existence of a second, concurrent service station having same service rates. To investigate the performance analysis of a state-interdependent working vacation queueing model, a [Markov model](#) was built. Some important performance measurements are derived from this model, which is very helpful in analysing the unique conditions that arise in settings like data voice transmission, computer communication systems, etc. The steady state solutions are derived and the specific uses are outlined for the model, a graphical analysis is provided to aid with comprehension by holding one parameter constant while changing others. By selecting $r=4$, $R=8$, and $v=20$, MATLAB calculations are employed to provide numerical examples.

Keywords: Markovian Queueing System, Vacation, Interdependent Arrival and Parallel Service Processes, Varying Arrival Rates, Bivariate Poisson Distribution.

1. Introduction

The queueing theory offers estimations for waiting times, the average number of customers, the length of the queue, and other variables. These predictions help us anticipate events and take action to shorten lines. In general, a queueing model's waiting queue lengthens as a result of either a sluggish service rate or a high arrival rate. The service rates are managed in a number of study studies to shorten the duration of the lines. About M/M/1, Srinivasa Rao et al. [2] have spoken.

In the earlier work, Aftab Begum and Maheswari (2002) [1] have analysed about M/M/c/ ∞ model. Levy

and Yechiali analysed the M/M/c queue with exponentially distributed vacation times [3]. Altman and Yechiali [4] presented a comprehensive analysis for M/M/c with vacations and impatient consumers. Here we assume that arrivals and services are correlated.

2. Model Description

Think about a queueing system for c-servers having limited capacity under the following presumptions. The arrival process and the service process are $\{X_1(t)\}, \{X_2(t)\}$ respectively are correlated and follow a bivariate Poisson process. (Figure 1-Figure 5)

$$P[X_1(t)=x_1, X_2(t)=x_2 | X_1(t) = x_1, X_2(t) = x_2]$$

$$= e^{-(\lambda_i + \mu - \varepsilon)t} \sum_{j=0}^{\min(x_1, x_2)} (\varepsilon t)^j [(\lambda_i - \varepsilon)t]^{x_1-j} [(\mu - \varepsilon)t]^{x_2-j} j! (x_1-j)! (x_2-j)! \\ = e^{-\left(\lambda_i + \mu - \varepsilon\right)t} \sum_{j=0}^{\min(x_1, x_2)} \left\{ \left(\varepsilon t\right)^j \left[\left(\lambda_i - \varepsilon\right)t\right]^{x_1-j} \left[\left(\mu - \varepsilon\right)t\right]^{x_2-j} \frac{1}{j! (x_1-j)! (x_2-j)!} \right\} \quad (1)$$

Where

$$x_1, x_2 = 0, 1, 2, \dots; \lambda_i, \mu, \varepsilon > 0, i = 0, 1, \dots, c-1, c, c+1, \dots, r-1, r, r+1, \dots, R-1, R, R+1, \dots; 0 \leq \varepsilon < \min(\lambda_i, \mu), x_1, x_2 = 0, 1, 2, \dots; \lambda_i, \mu_n > 0, i = 0, 1, \dots, c-1, c, c+1, \dots, r-1, r, r+1, \dots, R-1, R, R+1, \dots; 0 \leq \varepsilon < \min(\lambda_i, \mu),$$

It is presumptive that $c \ll r$. When the system has n components, the mean rate of service is defined as $\mu_n = \begin{cases} n\mu; & 0 \leq n < c \\ c\mu; & c \leq n \leq \infty \end{cases}$ $\mu_n = \begin{cases} n\mu; & 0 \leq n < c \\ c\mu; & c \leq n \leq \infty \end{cases}$

3. Steady State Equation

We notice that $P_{0,n}(0)P_{0,n}(0)$ and $P_{1,n}(0)P_{1,n}(0)$ exists when $0 \leq n \leq r$; $P_{0,n}(0)P_{0,n}(0), P_{1,n}(0)P_{1,n}(0), P_{0,n}(1)P_{0,n}(1)$ and $P_{1,n}(1)P_{1,n}(1)$ exists when $r+1 \leq n \leq R-1$ and $P_{0,n}(1)P_{0,n}(1) \& P_{1,n}(1)P_{1,n}(1)$ exists when $R \leq n \leq \infty$

$$(\lambda_0 - \varepsilon)P_{0,0}(0) = (\mu - \varepsilon)P_{1,1}(0) \left(\lambda_0 - \varepsilon \right) P_{0,0}(0) = (\mu - \varepsilon) P_{1,1}(0) \quad (2)$$

$$(\lambda_0 + v - \varepsilon)P_{0,n}(0) = (\lambda_0 - \varepsilon)P_{0,n-1}(0); (1 \leq n \leq R-1) \left(\lambda_0 + v - \varepsilon \right) P_{0,n}(0) = \left(\lambda_0 - \varepsilon \right) P_{0,n-1}(0); (1 \leq n \leq R-1) \quad (3)$$

$$(\lambda_1 + v - \varepsilon)P_{0,r+1}(1) = \left(\lambda_1 + v - \varepsilon \right) P_{0,r+1}(1) = 0 \quad (4)$$

$$(\lambda_1 + v - \varepsilon)P_{0,n}(1) = (\lambda_1 - \varepsilon)P_{1,n-1}(1); (r+2 \leq n \leq R-1) \left(\lambda_1 + v - \varepsilon \right) P_{0,n}(1) = \left(\lambda_1 - \varepsilon \right) P_{1,n-1}(1); (r+2 \leq n \leq R-1) \quad (5)$$

$$(\lambda_1 + v - \varepsilon)P_{0,R}(1) = (\lambda_1 - \varepsilon)P_{1,R-1}(1) \left(\lambda_1 + v - \varepsilon \right) P_{0,R}(1) = \left(\lambda_1 - \varepsilon \right) P_{1,R-1}(1) \quad (6)$$

$$(\lambda_1 + v - \varepsilon)P_{0,n}(1) = (\lambda_1 - \varepsilon)P_{1,n-1}(1); (R+1 \leq n \leq \infty) \left(\lambda_1 + v - \varepsilon \right) P_{0,n}(1) = \left(\lambda_1 - \varepsilon \right) P_{1,n-1}(1); (R+1 \leq n \leq \infty) \quad (7)$$

$$(\lambda_0 + \mu - 2\varepsilon)P_{1,1}(0) = 2(\mu - \varepsilon)P_{1,2}(0) + vP_{0,1}(0) \left(\lambda_0 + \mu - 2\varepsilon \right) P_{1,1}(0) = 2(\mu - \varepsilon)P_{1,2}(0) + vP_{0,1}(0) \quad (8)$$

$$(\lambda_0 + n\mu - (n+1)\varepsilon)P_{1,n}(0) = (\lambda_0 - \varepsilon)P_{1,n-1}(0) + (n+1)(\mu - \varepsilon)P_{1,n+1}(0) + vP_{0,n}(0); (2 \leq n \leq c-1) \left(\lambda_0 + n\mu - (n+1)\varepsilon \right) P_{1,n}(0) = \left(\lambda_0 - \varepsilon \right) P_{1,n-1}(0) + (n+1)(\mu - \varepsilon)P_{1,n+1}(0) + vP_{0,n}(0); (2 \leq n \leq c-1) \quad (9)$$

$$(\lambda_0 + c\mu - (c+1)\varepsilon)P_{1,n}(0) = (\lambda_0 - \varepsilon)P_{1,n-1}(0) + c(\mu - \varepsilon)P_{1,n+1}(0) + \left(\lambda_0 + c\mu - (c+1)\varepsilon \right) P_{1,n}(0) = \left(\lambda_0 - \varepsilon \right) P_{1,n-1}(0) + c(\mu - \varepsilon)P_{1,n+1}(0) +$$

$$c(\mu - \varepsilon)P_{1,n+1}(1) + vP_{0,n}(0); (c \leq n \leq r-1) c(\mu - \varepsilon)P_{1,n+1}(1) + vP_{0,n}(0); (c \leq n \leq r-1) \quad (10)$$

$$(\lambda_0 + c\mu - (c+1)\varepsilon)P_{1,r}(0) = (\lambda_0 - \varepsilon)P_{1,r-1}(0) + c(\mu - \varepsilon)P_{1,r+1}(0) + \left(\lambda_0 + c\mu - (c+1)\varepsilon \right) P_{1,r}(0) = \left(\lambda_0 - \varepsilon \right) P_{1,r-1}(0) + c(\mu - \varepsilon)P_{1,r+1}(0) +$$

$$c(\mu - \varepsilon)P_{1,r+1}(1) + vP_{0,r}(0) c(\mu - \varepsilon)P_{1,r+1}(1) + vP_{0,r}(0) \quad (11)$$

$$(\lambda_0 + c\mu - (c+1)\varepsilon)P_{1,n}(0) = (\lambda_0 - \varepsilon)P_{1,n-1}(0) + c(\mu - \varepsilon)P_{1,n+1}(0) + vP_{0,n}(0); \left(\lambda_0 + c\mu - (c+1)\varepsilon \right) P_{1,n}(0) = \left(\lambda_0 - \varepsilon \right) P_{1,n-1}(0) + c(\mu - \varepsilon)P_{1,n+1}(0) + vP_{0,n}(0);$$

$$(r+1 \leq n \leq R-2) (r+1 \leq n \leq R-2) \quad (12)$$

$$(\lambda_0 + c\mu - (c+1)\varepsilon)P_{1,R-1}(0) = (\lambda_0 - \varepsilon)P_{1,R-2}(0) + vP_{0,R-1}(0) \left(\frac{\lambda_0}{\mu - \varepsilon} + \frac{c}{\mu - \varepsilon} \right) P_{1,R-1}(0) = \left(\frac{\lambda_0}{\mu - \varepsilon} - \frac{1}{\mu - \varepsilon} \right) P_{1,R-2}(0) + vP_{0,R-1}(0) \quad (13)$$

$$(\lambda_1 + c\mu - (c+1)\varepsilon)P_{1,r+1}(1) = c(\mu - \varepsilon)P_{1,r+2}(1) + vP_{0,r+1}(1) \left(\frac{\lambda_1}{\mu - \varepsilon} + \frac{c}{\mu - \varepsilon} \right) P_{1,r+1}(1) = c(\mu - \varepsilon)P_{1,r+2}(1) + \{vP_{0,r+1}(1)\} \quad (14)$$

$$(\lambda_1 + c\mu - (c+1)\varepsilon)P_{1,n}(1) = c(\mu - \varepsilon)P_{1,n+1}(1) + (\lambda_1 - \varepsilon)P_{1,n-1}(1) + vP_{0,n}(1); \left(\frac{\lambda_1}{\mu - \varepsilon} + \frac{c}{\mu - \varepsilon} \right) P_{1,n}(1) = c(\mu - \varepsilon)P_{1,n+1}(1) + \left(\frac{\lambda_1}{\mu - \varepsilon} - \frac{1}{\mu - \varepsilon} \right) P_{1,n-1}(1) + vP_{0,n}(1);$$

$$(r+2 \leq n \leq R-1) (r+2 \leq n \leq R-1) \quad (15)$$

$$(\lambda_1 + c\mu - (c+1)\varepsilon)P_{1,R}(1) = c(\mu - \varepsilon)P_{1,R+1}(1) + (\lambda_1 - \varepsilon)P_{1,R-1}(1) \left(\frac{\lambda_1}{\mu - \varepsilon} + \frac{c}{\mu - \varepsilon} \right) P_{1,R}(1) = c(\mu - \varepsilon)P_{1,R+1}(1) + \left(\frac{\lambda_1}{\mu - \varepsilon} - \frac{1}{\mu - \varepsilon} \right) P_{1,R-1}(1)$$

$$+ (\lambda_0 - \varepsilon)P_{1,R-1}(0) + vP_{0,R}(1) + \left(\frac{\lambda_0}{\mu - \varepsilon} - \frac{1}{\mu - \varepsilon} \right) P_{1,R-1}(0) + vP_{0,R}(1) \quad (16)$$

$$(\lambda_1 + c\mu - (c+1)\varepsilon)P_{1,n}(1) = c(\mu - \varepsilon)P_{1,n+1}(1) + (\lambda_1 - \varepsilon)P_{1,n-1}(1) + vP_{0,n}(1); \left(\frac{\lambda_1}{\mu - \varepsilon} + \frac{c}{\mu - \varepsilon} \right) P_{1,n}(1) = c(\mu - \varepsilon)P_{1,n+1}(1) + \left(\frac{\lambda_1}{\mu - \varepsilon} - \frac{1}{\mu - \varepsilon} \right) P_{1,n-1}(1) + vP_{0,n}(1);$$

$$(n \geq R+1) (n \geq R+1) \quad (17)$$

Let

$$A = \lambda_0 - \varepsilon, B = \lambda_1 - \varepsilon, D = A + C, E = B + C, F = v; A = \frac{\lambda_0 - \varepsilon}{\mu - \varepsilon}, B = \frac{\lambda_1 - \varepsilon}{\mu - \varepsilon}, D = \frac{A}{A + C}, E = \frac{B}{B + C}, F = \frac{v}{\mu - \varepsilon}$$

From (2) we get

$$P_{1,1}(0) = AP_{0,0}(0) P_{1,1}(0) = AP_{0,0}(0) \quad (18)$$

From (3) we get

$$P_{0,n}(0) = D^n P_{0,0}(0); (1 \leq n \leq R-1) P_{0,n}(0) = D^n P_{0,0}(0); (1 \leq n \leq R-1) \quad (19)$$

From (4) and (5) we get

$$P_{0,n}(1)=0; (r+1 \leq n \leq R-1) P_{0,n}(1) = 0; (r+1 \leq n \leq R-1) \quad (20)$$

From (6) we get

$$P_{0,R}(1)=H^R-1P_{0,0}(0)=JP_{0,0}(0)P_{0,R}(1) = H^{R-1}P_{0,0}(0) = JP_{0,0}(0) \quad (21)$$

From (7) we get

$$P_{0,R}(1)=JH^n-RP_{0,0}(0); (R+1 \leq n \leq \infty) P_{0,R}(1) = JH^{n-R}P_{0,0}(0); (R+1 \leq n \leq \infty) \quad (22)$$

From (8) we get

$$P_{1,2}(0)=12(A^2+A-FD)P_{0,0}(0)P_{1,2}(0) = \frac{1}{2} \left(A^2 + A - FD \right) P_{0,0}(0) \quad (23)$$

Equation (9) can be written as

$$P_{1,n}(0)=1n[(A+(n-1))P_{1,n-1}(0)-AP_{1,n-2}(0)-FP_{0,n-1}(0)]; (3 \leq n \leq c) P_{1,n}(0) = \frac{1}{n} \left[(A + (n-1))P_{1,n-1}(0) - AP_{1,n-2}(0) - FP_{0,n-1}(0) \right]; (3 \leq n \leq c) \quad (24)$$

Equation (10) can be written as

$$P_{1,n}(0)=1c[(A+c)P_{1,n-1}(0)-AP_{1,n-2}(0)-FP_{0,n-1}(0)]; (c+1 \leq n \leq r) P_{1,n}(0) = \frac{1}{c} \left[(A + c)P_{1,n-1}(0) - AP_{1,n-2}(0) - FP_{0,n-1}(0) \right]; (c+1 \leq n \leq r) \quad (25)$$

Equation (11) can be written as

$$P_{1,r+1}(0)=1c[(A+c)P_{1,r}(0)-AP_{1,r-1}(0)-FP_{0,r+1}(0)-cP_{0,r+1}(1)]P_{1,r+1}(0) = \frac{1}{c} \left[(A + c)P_{1,r}(0) - AP_{1,r-1}(0) - FP_{0,r+1}(0) - cP_{0,r+1}(1) \right] \quad (26)$$

Equation (12) can be written as

$$P_{1,n}(0)=1c[(A+c)P_{1,n-1}(0)-AP_{1,n-2}(0)-FP_{0,n-1}(0)]; P_{1,n}(0) = \frac{1}{c} \left[(A + c)P_{1,n-1}(0) - AP_{1,n-2}(0) - FP_{0,n-1}(0) \right];$$

$$(r+2 \leq n \leq R-1) (r+2 \leq n \leq R-1) \quad (27)$$

Equation (13) can be written as

$$(A+c)P_{1,R-1}(0)=AP_{1,R-2}(0)+FP_{0,R-1}(0) (A+c)P_{1,R-1}(0) = AP_{1,R-2}(0) + FP_{0,R-1}(0) \quad (28)$$

From (14) we get

$$P_{1,r+2}(1) = 1c(B+c)P_{1,r+1}(1)P_{1,r+2}(1) = \frac{1}{c}(B+c)P_{1,r+1}(1) \quad (29)$$

Equation (15) can be written as

$$P_{1,n}(1) = 1c[(B+c)P_{1,n-1}(1) - BP_{1,n-2}(1)]; (r+3 \leq n \leq R)P_{1,n}(1) = \frac{1}{c} \left[(B+c)P_{1,n-1}(1) - BP_{1,n-2}(1) \right]; (r+3 \leq n \leq R) \quad (30)$$

Equation (16) can be written as

$$P_{1,R+1}(1) = 1c[(B+c)P_{1,R}(1) - BP_{1,R-1}(1) - AP_{1,R-1}(0)]P_{1,R+1}(1) = \frac{1}{c} \left[(B+c)P_{1,R}(1) - BP_{1,R-1}(1) - AP_{1,R-1}(0) \right] \quad (31)$$

Equation (17) can be written as

$$P_{1,n}(1) = 1c[(B+c)P_{1,n-1}(1) - BP_{1,n-2}(1)]; (R+2 \leq n \leq \infty)P_{1,n}(1) = \frac{1}{c} \left[(B+c)P_{1,n-1}(1) - BP_{1,n-2}(1) \right]; (R+2 \leq n \leq \infty) \quad (32)$$

Characteristics Of The Model

$$\begin{aligned} P(0) &= \sum_{n=0}^{\infty} [P_{0,n}(0) + P_{1,n}(0)]P(0) = \sum_{n=0}^{\infty} \{P_{0,n}(0) + P_{1,n}(0)\} \\ &= \sum_{n=r+1}^{\infty} P_{0,n}(0) + \sum_{n=1}^{\infty} 1cP_{0,n}(0) = \sum_{n=r+1}^{\infty} \{P_{0,n}(0) + \sum_{n=1}^{\infty} \frac{1}{c} \{P_{0,n}(0)\} + \sum_{n=c+1}^{\infty} rP_{1,n}(0) + \sum_{n=r+1}^{\infty} R-1P_{1,n}(0)\} \sum_{n=c+1}^{\infty} \{P_{1,n}(0)\} + \sum_{n=r+1}^{\infty} \{R-1\} \{P_{1,n}(0)\} \quad (33) \end{aligned}$$

$$\begin{aligned} P(1) &= \sum_{n=0}^{\infty} [P_{0,n}(1) + P_{1,n}(1)]P(1) = \sum_{n=0}^{\infty} \{P_{0,n}(1) + P_{1,n}(1)\} \\ &= \sum_{n=r+1}^{\infty} P_{0,n}(1) + \sum_{n=r+1}^{\infty} R P_{1,n}(1) + \sum_{n=R+1}^{\infty} P_{1,n}(1) = \sum_{n=r+1}^{\infty} \{P_{0,n}(1) + \sum_{n=r+1}^{\infty} \{R\} \{P_{1,n}(1)\} + \sum_{n=R+1}^{\infty} \{P_{1,n}(1)\} \quad (34) \end{aligned}$$

From the following normalising condition, the probability $P_{0,0}(0)P_{0,0}(0)$ that the system is empty may be determined.

$$P(0) + P(1) = 1 \quad (35)$$

Now,

$$L_s = L_{s0} + L_{s1}L_{s0} = L_{s0} + L_{s1} \quad (36)$$

where

$$Ls_0 = \sum_{n=1}^{R-1} R-1n P_{0,n}(0) + \sum_{n=1}^c n P_{1,n}(0) + \sum_{n=c+1}^r n P_{1,n}(0) + \sum_{n=r+1}^{R-1} R-1n P_{1,n}(0) L_{\{s_0\}} = \\ \sum_{n=1}^{R-1} \{n P_{0,n}(0)\} + \sum_{n=1}^c \{n P_{1,n}(0)\} + \sum_{n=c+1}^r \{n P_{1,n}(0)\} + \sum_{n=r+1}^{R-1} \{n P_{1,n}(0)\}$$

and

$$Ls_1 = \sum_{n=r+1}^{\infty} n P_{0,n}(1) + \sum_{n=r+1}^{\infty} n P_{1,n}(1) L_{\{s_1\}} = \sum_{n=r+1}^{\infty} \{n P_{0,n}(1)\} + \sum_{n=r+1}^{\infty} \{n P_{1,n}(1)\}$$

By using Little formula we get

$$Ws = Ls \lambda^- W_{\{s\}} = \frac{L_{\{s\}}}{\overline{\lambda}},$$

Where

$$\lambda^- = \lambda_0 P(0) + \lambda_1 P(1) \overline{\lambda} = \lambda_{00} P(0) + \lambda_{11} P(1) \quad (37)$$

5. Arithmetic Example And Graph

For various values of $\lambda_0, \lambda_1, \mu, \epsilon, c$ the ideals of $P_{0,0}(0), P(0), P(1), L_{\{s\}}$ and Ws are calculated using MATLAB and been enumerated below Table 1 and Table 2

Let $r=4, R=8, v=20$

Table 1: When $c=1$

λ_0	λ_1	μ	ϵ	$P_{0,0}(0)$	$P(0)$	$P(1)$	$L_{\{s\}}$	Ws
5	2	3	0	0.0113	0.5725	0.4275	19.3858	5.2149
6	2	3	0	0.0044	0.5221	0.4779	20.4470	5.0013
7	2	3	0	0.0019	0.4945	0.5055	21.0492	4.7064
8	2	3	0	9.3052e-004	0.4804	0.5196	21.3832	4.3832
9	2	3	0	4.8969e-004	0.4745	0.5255	21.6066	4.0601

λ_0	λ_1	μ	ϵ	$P_{0,0}(0)P_{\{0,0\}}(0)$	$P(0)$	$P(1)$	L_s	W_s
10	4	5	0.5	0.0030	0.4836	0.5164	20.9810	3.0401
10	5	5	0.5	0.0026	0.4287	0.5713	21.5505	3.0168
10	6	5	0.5	0.0023	0.3755	0.6245	22.1027	2.9463
10	7	5	0.5	0.0020	0.3259	0.6741	22.6173	2.8350
10	8	5	0.5	0.0017	0.2812	0.7188	23.0818	2.6957
6	2	2	0	4.1810e-004	0.3935	0.6065	22.3551	6.2548
6	2	3	0	0.0044	0.5221	0.4779	20.4470	5.0013
6	2	4	0	0.0197	0.6478	0.3522	18.1497	3.9531
6	2	5	0	0.0551	0.7592	0.2408	15.7883	3.1345
6	2	6	0	0.1134	0.8449	0.1551	13.7630	2.5584
8	6	5	0	0.0101	0.4547	0.5453	20.6797	2.9930
8	6	5	0.3	0.0089	0.4432	0.5568	20.8661	3.0300
8	6	5	0.5	0.0082	0.4351	0.5649	20.9971	3.0562
8	6	5	0.7	0.0074	0.4266	0.5734	21.1337	3.0837
8	6	5	1	0.0063	0.4131	0.5869	21.3493	3.1275



Figure.1: (Finding Ls and Ws by varying λ_0 and keeping other parameters fixed)

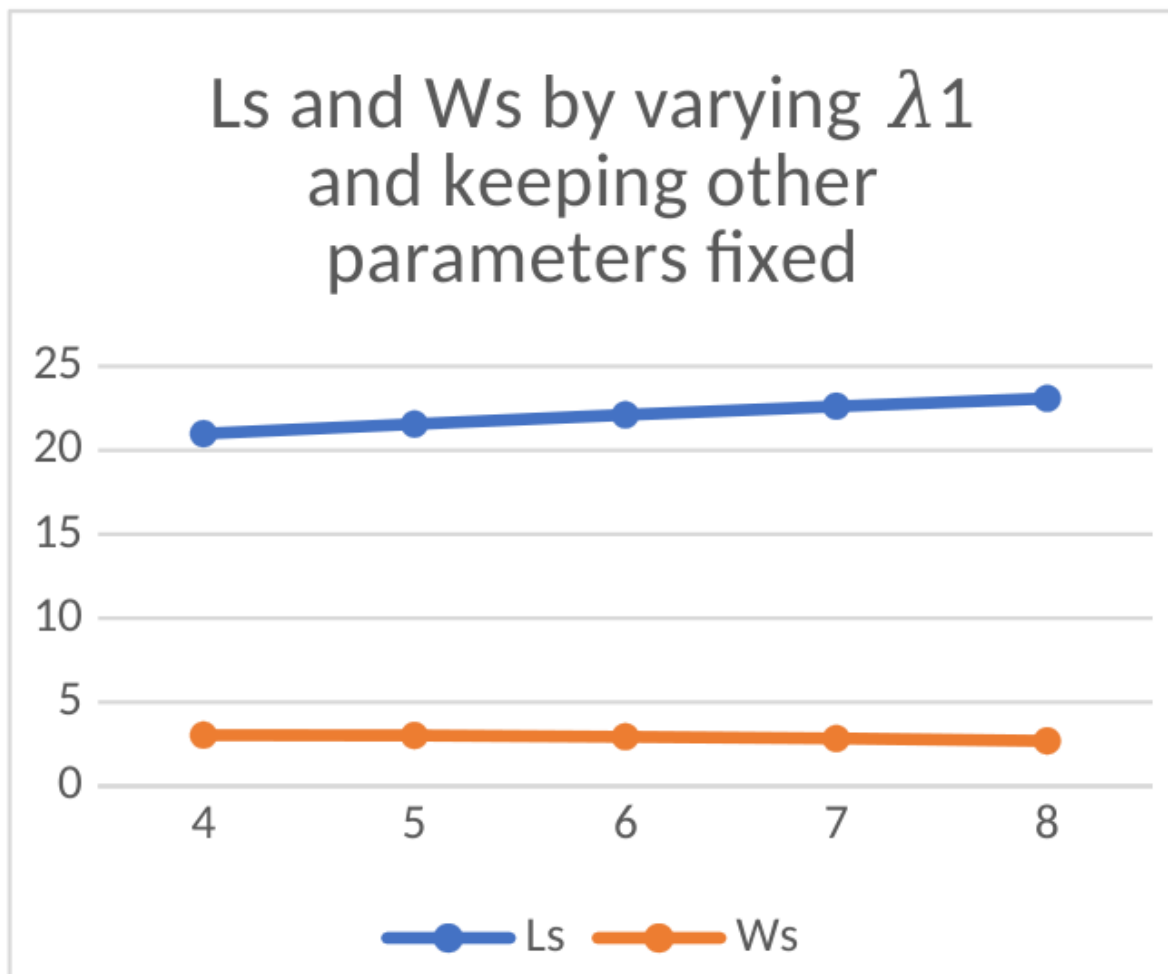


Figure.2: (Finding Ls and Ws by varying λ_1 and keeping other parameters fixed)

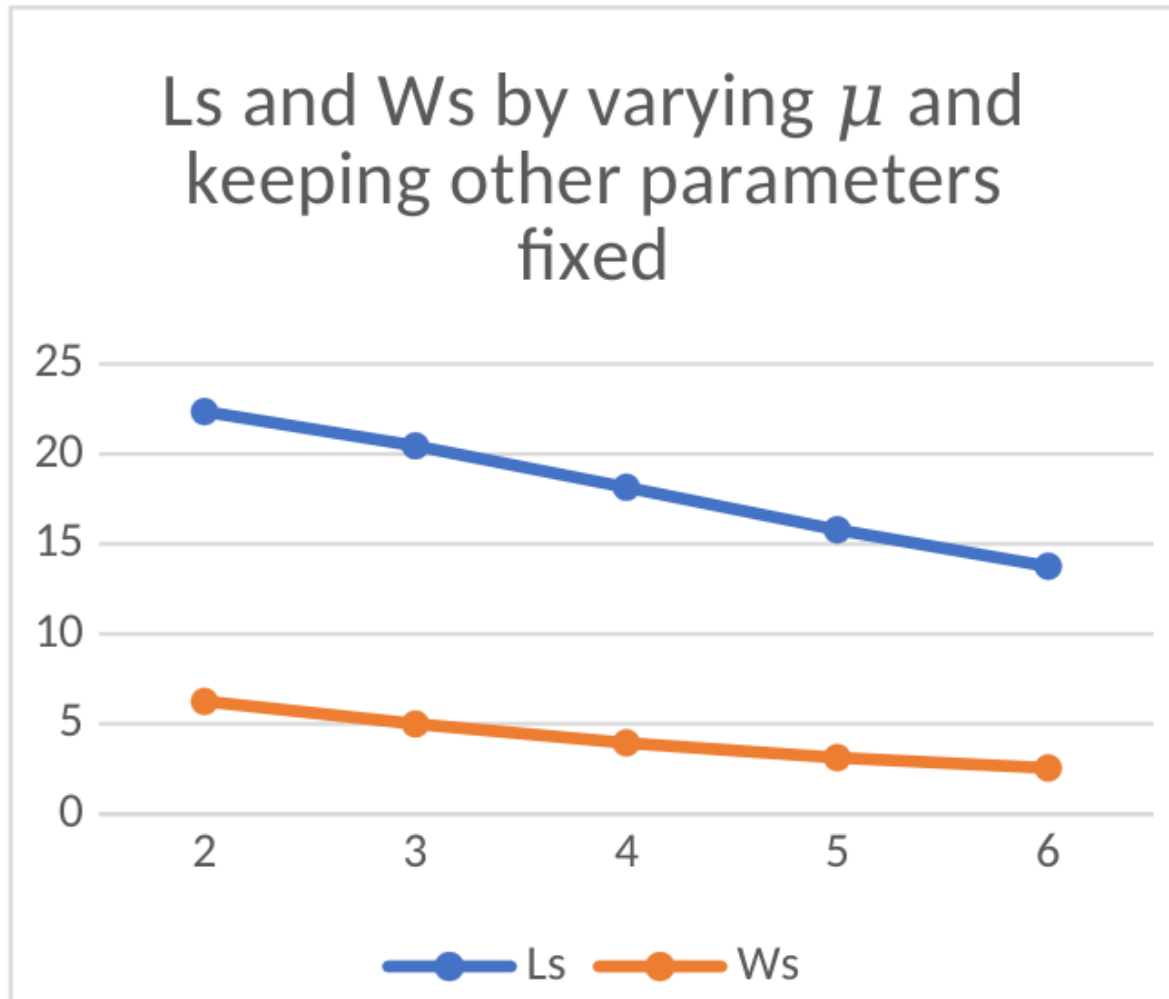


Figure.3: (Finding Ls and Ws by varying μ and keeping other parameters fixed)

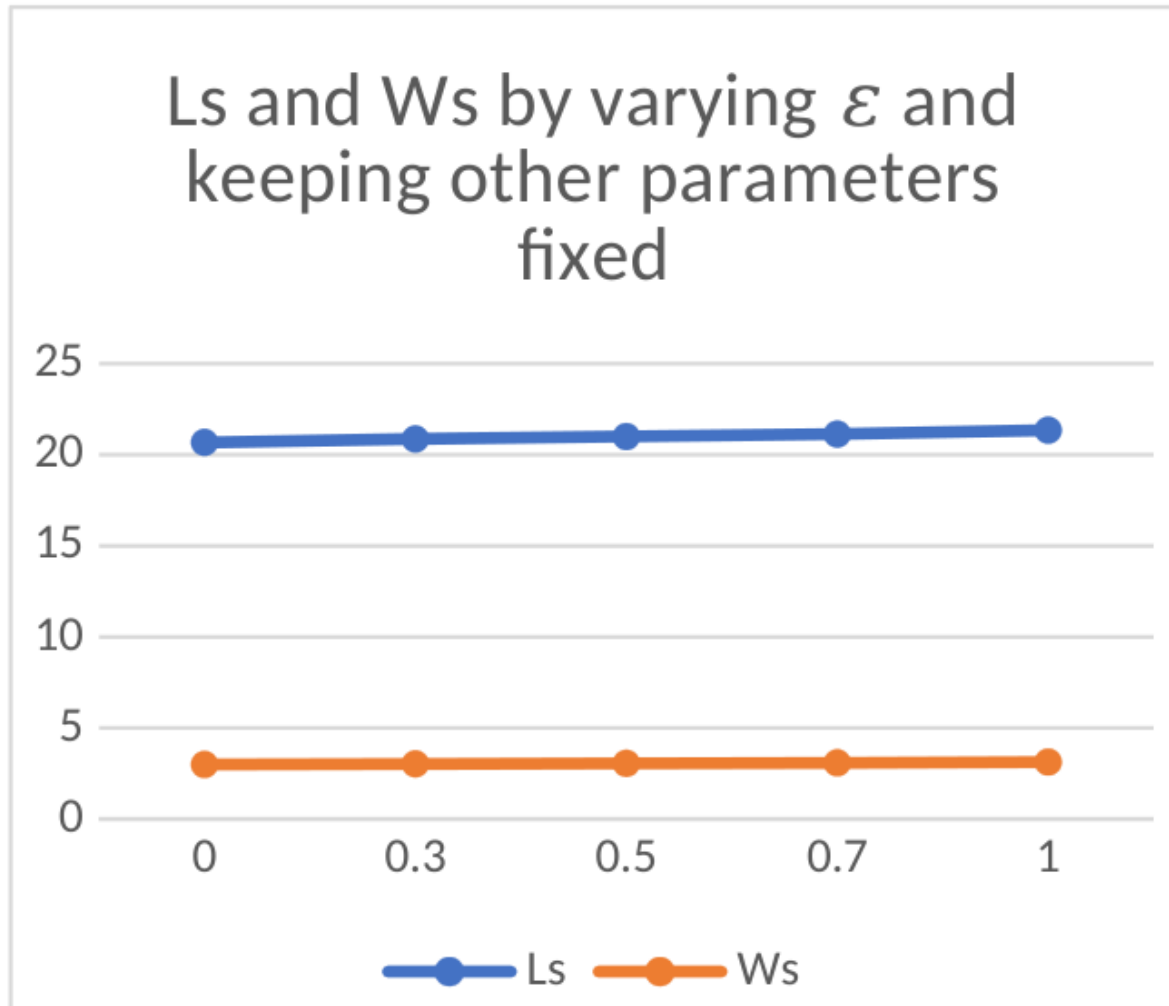


Figure.4: (Finding Ls and Ws by varying ε and keeping other parameters fixed)

Table 2: When $c=2$

λ_0	λ_1	μ	ε	$P_{0,0}(0)$	$P(0)$	$P(1)$	L_s	W_s
5	2	3	0	0.1245	0.8400	0.1600	10.2298	2.2633
6	2	3	0	0.0682	0.7153	0.2847	12.7184	2.6163
7	2	3	0	0.0388	0.5730	0.4270	15.0815	3.0999
8	2	3	0	0.0233	0.4189	0.5811	17.2227	3.8160
9	2	3	0	0.0148	0.2493	0.7507	19.2021	5.1269

λ_0	λ_1	μ	ϵ	$P_{0,0}(0)P_{\{0,0\}}(0)$	$P(0)$	$P(1)$	L_s	W_s
10	4	5	0.5	0.0524	0.6567	0.3433	13.9572	1.7578
10	5	5	0.5	0.0506	0.6342	0.3658	14.3701	1.7587
10	6	5	0.5	0.0487	0.6101	0.3899	14.8115	1.7548
10	7	5	0.5	0.0466	0.5847	0.4153	15.2777	1.7452
10	8	5	0.5	0.0445	0.5582	0.4418	15.7636	1.7292
6	2	2	0	0.0137	0.2237	0.7763	19.8620	6.8615
6	2	3	0	0.0682	0.7153	0.2847	12.7184	2.6163
6	2	4	0	0.1640	0.8968	0.1032	9.2185	1.6499
6	2	5	0	0.2760	0.9608	0.0392	7.9380	1.3585
6	2	6	0	0.3884	0.9837	0.0163	7.7156	1.3001
8	6	5	0	0.1253	0.8276	0.1724	10.8548	1.4180
8	6	5	0.3	0.1176	0.8128	0.1872	11.0929	1.4547
8	6	5	0.5	0.1121	0.8014	0.1986	11.2817	1.4839
8	6	5	0.7	0.1063	0.7887	0.2113	11.4984	1.5175
8	6	5	1	0.0971	0.7664	0.2336	11.8846	1.5777

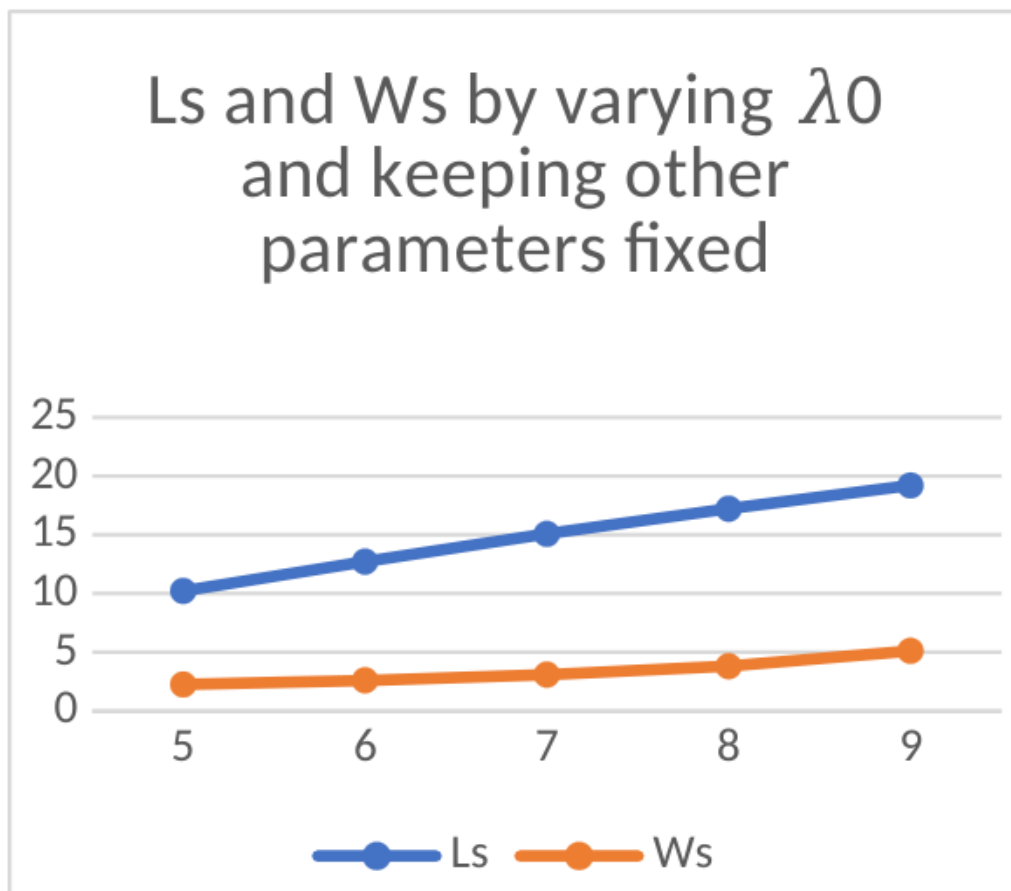


Figure.5: (Finding Ls and Ws by varying λ_0 and keeping other parameters fixed)

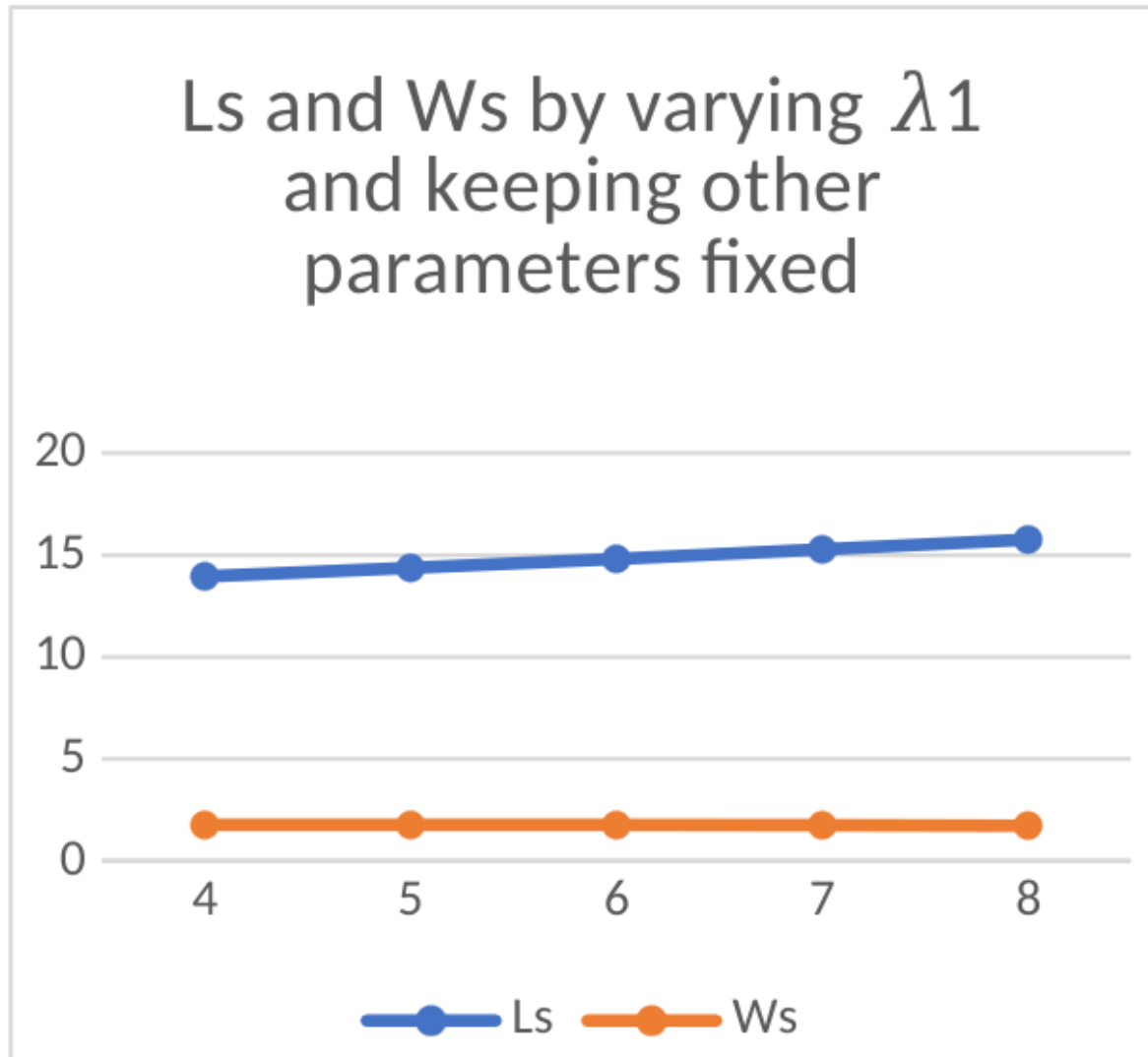


Figure.6: (Finding Rs and Ws by varying λ_1 and keeping other parameters fixed)

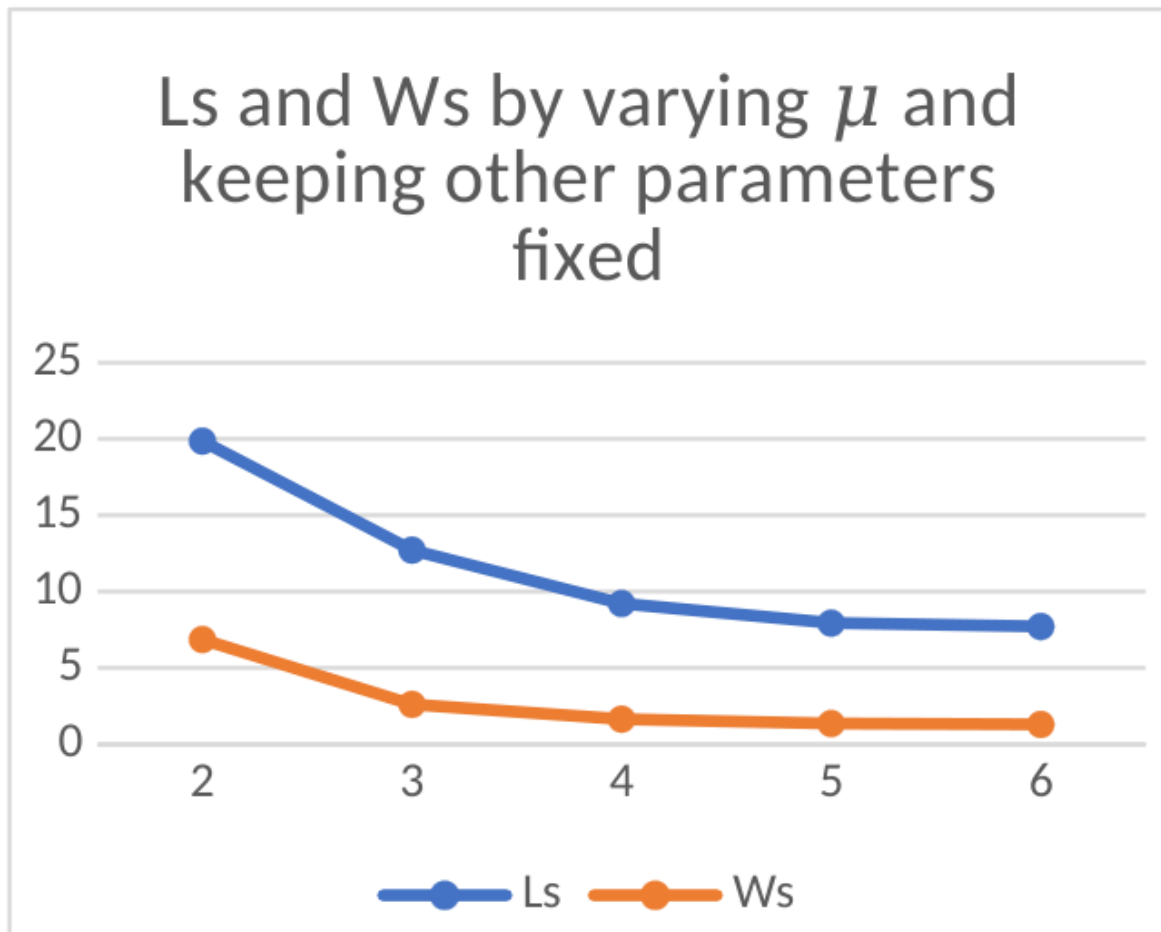


Figure.7: (Finding Ls and Ws by varying μ and keeping other parameters fixed)

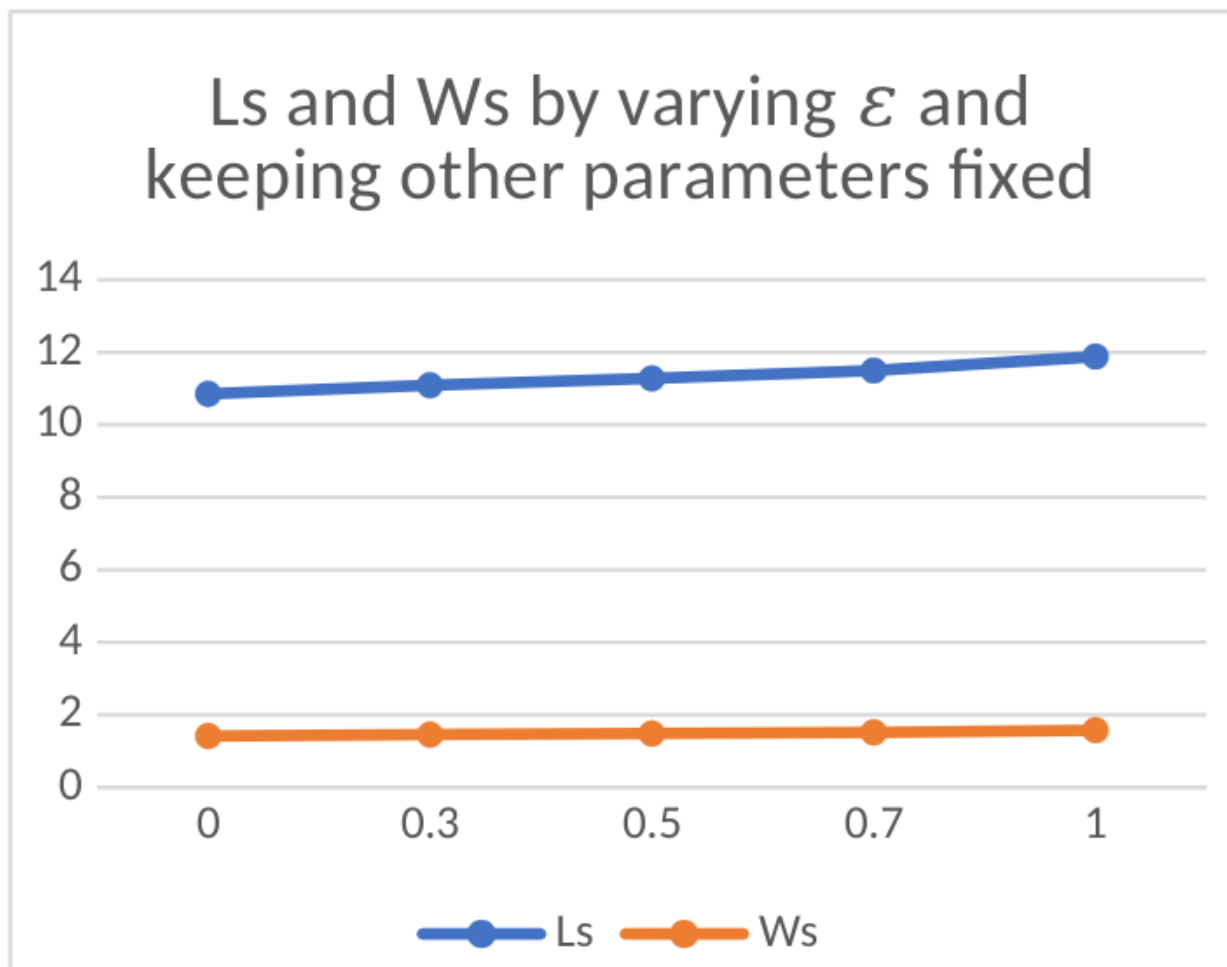


Figure.8: (Finding Ls and Ws by varying ε and keeping other parameters fixed)

6. Conclusion

The reported tables show that increasing the service rate, with the other parameters fixed, decreases both Ls and Ws. Increasing the dependence parameter epsilon increases both quantities in the displayed examples. Increasing the number of parallel servers generally reduces Ls and Ws in these examples, although the $c=2$ waiting time exceeds the $c=1$ value for the row with $\lambda_0=6$, $\lambda_1=2$, $\mu=2$ and $\varepsilon=0$. Increasing an arrival rate increases Ls in the displayed examples; the direction of the change in Ws depends on the arrival-rate parameter and server configuration. These observations describe the reported numerical tables and do not resolve the model and stability issues identified in the accompanying review.

This paper incorporates the original models as special cases. For instance, when $v=0$ this model narrows down to the M/M/c/ ∞ interdependent queuing model that allows for varying arrival rates [1].

When λ_0 tends to λ_1 and $\varepsilon=0$, this model shrinks to the M/M/c/ ∞ queueing system's analysis using vacations [5]. When λ_0 tends to λ_1 , $\varepsilon=0$ and $v=0$, this model narrows down to the conventional M/M/c/ ∞ queueing model.

Data Availability Statement

Not Applicable

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No fund received for this project

Conflicts Of Interest

The authors declare that they have no conflict of interest.

EthICAL Approval And Human Participation

No ethics approval is required.

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