Graph Theoretical Notation of the Network

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Abstract

We use complex network theory to study the differences between the countries' international relationship concepts in Formal Interstate Alliance (FIA) and Dyadic Militarized Interstate Disputes (MID). Seven IR networks were identified. Six of these networks are from the FIA, and one is from the MID.

Since community detection is computationally challenging, using modularity to calculate real-world social network datasets, we provide a simple modularity algorithm and verify its performance with real social network datasets. The idea is to identify emerging trends and use network techniques to examine the IR concept. Additionally, we are interested in identifying the most influential, central, as well as active nodes using sociometric analyses. We analyzed these IR networks' structure and communities and found significant differences in Formal Interstate Alliance (FIA) compared with Dyadic Militarized Interstate Disputes (MID). In FIA, the countries make alliances because of the presence of various types of relationships, such as defense pacts, neutrality or non-aggression treaties, and entente agreements. These kinds of relationships are called positive relationships. These differences made up a topological structure quite different at different international levels. We also found differences in the network characteristics. Once these differences are understood, the topological structure of the IR network and the communities shaped in FIA could be predicted if we know the total number of nodes and the ties. However, this discovery implies that IR is a dynamic concept that produces several changes in the IR network structure and the way that countries make groups of relationships; it provides the opportunity to give analytic support to empirical studies.

Introduction

Social, technological, and information systems can usually be defined as complex networks with a topology of interconnected nodes combining organization and randomness. Network theory provides a way to study complex systems effectively, such as metabolic networks, protein-protein interaction networks, and transportation networks. It has also been found that these complex networks own many common topological properties. For example, many complex networks have been found to have community structure or modular structure. Social networks consist of different kinds of relationships. Two representative relationships between agents are positive (friendship) and negative (hostility). For example, nations may be formal allies like Japan and the United States or may have military conflicts

like South Korea and North Korea. For these kinds of relationships, social balance theory in psychology and sociology states that when a social triad consisting of three agents is considered, the triad probably consists either of three positives or of one positive and two negatives [24, 25].

One of the key research in this area is community structures. A community is a bunch of nodes in a network with similar properties or more connections. Detection of a group of nodes in a network is a goal to identify strongly connected components by separating sparse connections. Community detection is a partitioning procedure that attempts to find relatively dense sets of nodes that are relatively sparsely connected to the other sets. Therefore, numerous algorithms have been proposed to find reasonably good partitions reasonably fast. In recent years, this search has attracted much interest in fast algorithms due to the increasing availability of large network data sets and the influence of networks on everyday life.

There is a renewed interest in communities and their structures, as well as in the algorithms to describe them [15, 22, 23]. Computational efficiency is one of the main issues that must be taken into account when dealing with the problem of finding communities in real-world social networks [18-21]. Yet researchers have been trying to get reasonable computations for clustering using modularity.

In this paper, we are interested in identifying how communities appear in real-world social network datasets such as formal interstate alliances and dyadic militarized Interstate dispute datasets. We take into international relationships (IR) within the country's network. To do this we employ Louvain's algorithm using modularity approach [13], for all seven networks. This algorithm is ideal for finding large-scale communities because of its analytical base, which clearly shows network division into communities. We consider this study important because it allows us to provide an analytic and computational basis to the empirical studies, thus providing the lines of research for applied physics.

The outline of the rest of the paper is as follows:

We first present the background of networks and community detection. Next, in the methodology section, we discuss the data sets and the communities analyzed using modularity in the networks. Next, in the results section, we discuss the implications of analyzing the networks using modularity. Then, we describe the discussion of the results, and last, it is followed by the conclusions section.

Background

Many networks of interest in the sciences, including social networks, computer networks, and metabolic and regulatory networks, are found to divide naturally into communities or modules. Community structure detection is a data analysis technique used to analyze the structure of large-scale network data sets, such as social networks, internet and web data, or biochemical networks [1]. Community structure methods usually undertake the network of interest to be divided naturally into subgroups, and the experimenter's job is to find those groups. The number and size of the groups are thus determined by the network itself and not by the experimenter.

2.1. Graph Theoretical Notation of the Network

A complex network is mapped to the graph G(V, E), where V represents the node set and E represents the edge set. The cardinality (order) of V and E is shown by n and m, respectively. If v is the subset of V and e is the subset of E, then it is said to be a subnetwork of a network C(v, e).

2.2. Community detection

In the broad applications of community detection in the area of complex network analysis, there are different definitions presented for the community, among which two descriptions have become popular among data scientists. (i) A set of nodes with similar properties, called a community [11]. (ii) A community is defined as a set of nodes that has more edges in a group rather than the other nodes in the rest of the graph [12]. A broad overview of the community detection methods can be found in [14-17].

2.2.1. Modularity

Modularity has often been used as a quality metric to result in good partitions formed by these methods by comparing the number of links within and between communities. The range of value of modularity is from -1 to 1. The modularity value near 1 indicates the good quality of partitions, while the value near -1 indicates the bad quality of partitions. Researchers have been exploring to achieve reasonable approximations for modularity.

2.3. Community Detection Approaches

Uncovering community structure helps to understand network functionality. However, community detection is computationally inflexible in large-scale networks. No accepted solution for community detection exists yet, and many techniques for optimizing community detection have been introduced in the scientific literature.

In this section, we present an overview of the six state-of-the-art groups of community detection algorithms. We provide techniques for the identification of static, dynamic, disjoint, and overlapping communities. A comprehensive overview of the community detection techniques can be found in [14, 15, 17].

2.3.1. Traditional Community Detection Techniques

We focus on traditional techniques of community detection.

2.3.1.1. Graph partitioning

It splits the graph into g clusters of predefined size, such that the number of links in a cluster is denser than the number of edges between the clusters (15). The spectral Bisection method is a well-known example of a graph partitioning technique (27) and the Kernighan-Lin algorithm (26).

2.3.1.2. Hierarchical clustering

The graphs may hold a hierarchical structure, where each community may be a collection of small groups at different levels (15; Friedman et al. 2001). In such cases, hierarchical clustering techniques may be used to classify the multilevel community structure of the graph. Hierarchical grouping techniques are based on the node similarity measure. As it does not need a predefined size or number of communities. Dendrograms can represent them better. Hierarchical clustering approaches can be divided into two classes.

- Agglomerative algorithms

It is a bottom-up method as, at the start, it studies each node as a separate cluster and iteratively merges them based on high similarity, resulting in a unique community.

- Divisive algorithms

It is a top-down method as, at the start, it considers the whole network as a single cluster and iteratively divides it by eliminating links joining nodes with low similarity and ends up with unique communities.

2.3.1.3. Partitional clustering

Partitional clustering [15] divides a dataset into a predefined number of k non-overlapping clusters. The aim is to partition the data points into k clusters in order to minimize /maximize the cost function based on dissimilarity measures between nodes. Some of the frequently used cost functions are minimum k median, k-clustering sum, k-clustering, and k-center. Examples of partial clustering methods include k-mean clustering and fuzzy k-mean clustering. In fuzzy k-mean, clustering is one node that may fit into several clusters.

2.3.1.4. Spectral clustering

Spectral clustering includes all methods using eigenvectors of matrices to split the data points based on the pairwise similarity between them [15]. Examples contain a Laplacian spectral clustering method of Fiedler [28] and Donath

2.3.1.5. Divisive algorithms

It eliminates inter-cluster edges in a network based on low similarity to separate communities from each other [31]. The key examples of this type include the Girvan-Newman algorithm [30], where edges eliminated are iteratively based on the edge-betweenness score, and the Radicchi et al. technique [32], where edges are eliminated iteratively based on the edge clustering coefficient.

2.3.2. Modularity Optimisation-Based Community Detection Techniques

It presents network community detection techniques based on modularity optimization. Modularity is a measure of networks or graphs that was designed to measure the power of division of a network into modules, or it is the quality that approximates the communities. The larger the modularity value, the better the partition.

2.3.2.1. Greedy techniques

- Greedy method of Newman

Newman's greedy search algorithm [33] was the first algorithm recommended for modularity optimization. It is an agglomerative method, where originally, each node belongs to a discrete module, then they are fused iteratively based on the modularity gain. On sparse networks, it has a time complexity of O(n)3 on sparse networks.

- Fast Greedy algorithm (CNM)

It is the fast version of Newman's algorithm [33], implemented by efficient data structures on sparse networks. It has a computational complexity of O(nlog2n).

- Blondal's Louvain algorithm

Louvain [34] is a heuristic greedy algorithm for discovering communities in complex weighted networks. It is also based on modularity optimization. It allocates different communities to each vertex, one per vertex. It iteratively combines the nodes based on the gain of modularity. If there is no gain, the node remains in its community. The process is repeated until no more improvement is possible. It then rebuilds the network in the way that communities identified in the first phase are replaced by super nodes. Its time complexity is given like O(nlogn).

2.3.2.2. Simulated Annealing (Kirkpatrick et al. 1983)

It is a discrete stochastic method used to optimize the given objective function globally. Guimer`a et al. [35] have used a simulated annealing-based modularity optimization method. Primarily, it divides the network into random clusters. The optimization is based on local and global changes. Local moves move a node randomly from one cluster to another based on modularity gain. Global moves consist of dividing and merging clusters.

2.3.2.3. Extremal Optimization

Boettcher et al. [36] have calculated extremal optimization as a general-purpose heuristic search method for physical and combinatorial optimization problems. It is proposed to get accuracy comparable with genetic algorithm and simulated annealing. It emphasizes the optimization of local variables. Duch et al. in [37] have used it for modularity optimization. It allocates fitness to each node; the fitness value is gained by taking the ratio of the local modularity of the node by its degree. It changes an individual solution within a single configuration and makes local variations. It starts by randomly dividing the network into two clusters of the same order. It iteratively moves vertices with the lowest fitness to the other partitions. After moving, the clusters change, that’s why it recalculates local fitness of many nodes. The procedure repeats until an optimal value of global modularity is obtained.

2.3.2.4. Spectral Optimization

Spectral optimization is the use of eigenvectors and eigenvalues of the modularity matrix for modularity optimization [15]. This optimization is pretty fast. Examples of the spectral optimization of modularity are [38, 12].

2.3.2.5. Evolutionary algorithms

Evolutionary algorithms are a branch of metaheuristic optimization algorithms based on artificial intelligence. They are known for their real, local learning and global searching competencies. These approaches are largely divided into two classes based on single and multi-objective optimization. Examples of the first kind include MAGA-Net [39], MLAMA-Net [40] etc. Examples of the second category include MOEA/D [39].

2.3.3. Overlapping community detection techniques

Most of the vertices in real-world networks may concurrently belong to multiple communities. Traditional community detection methods fail to recognize overlapping communities. Clique

percolation is the most well-known technique for uncovering overlapping communities in networks. It is based on the idea that cliques are more likely to be shaped from internal edges that are densely connected than from external edges that are sparsely connected. The communities are made up of k-cliques that refer to the complete subgraphs with k vertices. Two cliques are said to be adjacent if they share a k − 1 node. The k-clique community is the giant component shaped by all the adjacent k-cliques connected as a k-clique series. Other examples of this category contain top graph clusters [41].

2.3.4. Dynamic Community Detection Algorithms

This focuses on community detection methods in dynamic networks like Twitter, Facebook, and LinkedIn. These techniques study the community assignment of the changed or new nodes during temporal updates in the network.

2.3.4.1. Potts model

The Potts model is one of the renowned models used in statistical physics. It proves a system of spins that can be in diverse states. The interaction among neighboring turns may be ferromagnetic or antiferromagnetic. Potts spin variables can be mapped to the nodes of the graph having community structure. From interactions between neighboring spins, it is plausible that community structure may be recognized from like-valued spin clusters of the system, as there may be more interactions in the community and fewer interactions outside the community.

2.3.4.2. Random walk

The Random Walk is adopted to find communities. In a random walk, the walker initiates walking inside a community from a node, and at each step, it moves to the neighboring node selected randomly and consistently. The walker takes a long time inside the dense communities because of the high density and multiple paths. The PageRank algorithm is an example of the most general techniques based on random walks.

2.3.4.3. Diffusion Community

“A diffusion community in a complex network is a set of nodes that are gathered together by the propagation of the same property, action, or information in the network. Label and dynamic node coloring are examples of this category.

2.3.4.4. Synchronization

Synchronization is an emerging method that has interest from different fields. It happens in interacting units and is convincing in nature, technology, and society. The system units remain in the same or similar states over time in a synchronized state. Synchronization is also used in community discovery in networks. If oscillators are located at nodes with random initial phases and interact with nearest neighbors, oscillators going to the same partition synchronize first, whereas longer time is obligatory for full synchronization. If evolution time of the process is followed, then states with synchronized communities of nodes can be much stable and prolonged, therefore can be known easily.

3. Methodology

In this work, we have used undirected international relationship (IR) networks where six formal interstate alliance networks and one military dispute network are constructed. A method finds partitions, clusters, or communities based on network structural properties. For this purpose, the Louvain algorithm strategy of community detection is followed. The Louvain algorithm takes each node as a community. By comparing each node with its neighbors, it resolves to fuse communities with the maximum possible gain in modularity. A question arises: is it possible to observe the changes in the concept of international relationships (IR) through the IR networks, their properties, and communities? It is an interesting domain that motivates researchers to work on it progressively.

3.1. Complex Network Tools: An Overview

An effective set of mathematical, statistical, and computational tools for social realities network analysis and modeling is available. Every tool has a different design and computational model; one size does not fit all. Some are specialized, some are common, some are single threaded, some are multi-threaded, some are having shared memory, some are having distributed memory. Network scientists use these tools for network construction, visualization, simulation, property extraction, pattern recognition, community detection, and complex queries. Some of the usually used network analysis and visualization tools include Pajek, MATLAB, R, Gephi, Visone, Wolfram Mathematica, NodeXL CiteSpace, etc.

3.2. Tool Used for Analysis and Visualization

Gephi is an open-source software package written in Java on the NetBeans platform, used for network analysis and visualization [4]. Gephi has been used in several research projects in academia, journalism, and elsewhere, for example, in visualizing the global connectivity of New York Times

content [5]and investigating Twitter network traffic during social unrest[6,7] along with more traditional network analysis topics [8]. Gephi is broadly used within the digital humanities [9], literature, political sciences, etc., a community where many of its developers are involved. Gephi inspired the LinkedIn InMaps [10] and was also used for the network visualizations for Truthy.

3.2. Data Description

In this work, we use Correlates of War Formal Interstate Alliance Dataset (v4.1), as a positive relationship provided by Gibler and Press [2] and available publicly. This dataset defines alliance relationships between countries according to different types, such as defense pacts, neutrality or non-aggression treaties, and entente agreements. The negative relationships between countries are provided by the Dyadic Militarized Interstate Disputes Dataset Version 2.0, which is made by Maoz [3] and is available publicly.

3.3 The Louvain method for community detection in large networks

The Louvain method is a simple, efficient, and easy-to-implement method for classifying communities in large networks. The method has been used in many different types of networks and for sizes up to 100 million nodes and billions of links. The method reveals hierarchies of communities and allows one to zoom within communities to discover sub-clusters, sub-sub-clusters, etc. For detecting communities in large networks, It is today one of the most broadly used techniques [13].

Louvain [10] is a heuristic greedy algorithm for identifying communities in complex weighted graphs. It is based on modularity optimization, too. It allocates different communities to each vertex, one per vertex. It iteratively combines the nodes based on the gain of modularity. If there is no gain, the node remains in its community. The procedure is repeated until no more development is possible. It then reconstructs the network in the way that communities recognized in the first phase are substituted by super nodes. The time of this algorithm complexity is O(nlogn).

3.4. Network Characteristics

In this work, undirected networks of IR are used. These seven networks show the dyad relationship between the two countries. The networks are constructed using the Gephi tool. The general characteristics of the networks are described in Table 1, such as nodes, No. Of edges, average degree, average weighted degree, network diameter, density, average clustering coefficient, average path length, number of weakly connected components, etc; of the Alliance By Directed, Alliance By Directed (Yearly), Alliance By Dyad, Alliance By Dyad (Yearly), Alliance By Member, Alliance By Member (Yearly), Dyadic MID networks respectively. Table 2 shows different centrality metrics such as Hubs, Authority, PageRank, Betweenness Centrality, Closeness Centrality, Eigenvector Centrality,

Eccentricity, and Harmonic closeness centrality of the seven networks.

Table 3 indicates the community detection of the Alliance By Directed, Alliance By Directed (Yearly), Alliance By Dyad, Alliance By Dyad (Yearly), Alliance By Member, Alliance By Member (Yearly), and Dyadic MID networks, respectively, using modularity. The Louvain community detection technique is used, keeping the resolution equal to 1, using the weights, and keeping it randomized. Different modularity classes are displayed, showing the percentage by which the countries are in alliance or dispute relationships, making densely connected networks.

4. Results and discussion

4.1. Alliance By Directed and Directed (Yearly) Network Analysis

The Alliance By Directed has 859 and Directed (Yearly) has 899 nodes, respectively. Seven communities are detected in both networks, while the modularity value of AB directed is 0.279, which is much better than that of the other one. The countries' highly densely connected relationship makes the alliance. Figures 1 and 2 and Table 3 show the community detection graphs, modularity class distribution graph, and number of communities plus the modularity value of the networks, respectively.

4.2. Alliance By Dyad and Dyad (Yearly) Network Analysis

The Alliance By Dyad has 860 and Dyad (Yearly) has 899 nodes, respectively; seven and six communities are detected in both networks, respectively, while the modularity value of the AB dyad is 0.258, which is much better than the other one. It is the countries' highly densely connected relationship that makes the alliance. Figures 3, 4, and Table 3 show the community detection graphs, modularity class distribution graph, and no. of communities plus modularity value of the networks, respectively.

4.3. Alliance By Member and Member (Yearly) Network Analysis

The Alliance By member having 872 and member (Yearly) having 912 nodes respectively, eleven and eight communities are detected in both networks respectively, while the modularity value of member (Yearly) is 0.236, which is much better than the other. It is the countries' highly densely connected relationship that makes the alliance. Figures 5, 6, and Table 3 show the community detection graphs,

modularity class distribution graph, and no. of communities plus modularity value of the networks, respectively.

4.4. Dyadic MID Network Analysis

The Dyadic MID network has 3301 nodes, with 11 communities detecting having a modularity value of 0.143, depicting the disputed relationship of the network. Consequently, it is clear and obvious that the IR concept is dynamic as there is a change in network topologies, structures, properties, and as well as communities that are quite different from each other while making different kinds of positive and negative relationships.

4.5. An Analysis of an Example of a Social Friendship Network of Elementary School (E1)

This analysis was conducted using the adjacency matrices obtained for the network that is depicted in Table 4. The properties that characterize this social network are also displayed. Louvain Algorithm and VOS clustering algorithm are used to find the community structure of the network and the relationship between the friendship concept and that structure is also investigated. To compare the community detection in the same network (E1) with both different algorithms, it results that the VOS clustering algorithm results in 0.507 is much better based on modularity, and eight communities are detected, while 6 communities are detected in the Louvain method with modularity value of 0.433 that is less than the VOS clustering algorithm. Pajek software is used for network analysis, community detection, and visualization of this elementary school friendship network (E1). Figure 8 depicts the friendship Network of E1, the extraction of communities according to the Louvain Method, and its community detection.

Table 1. Principal Characteristics of International Relationship

Network Characteristics	Formal Alliances (v4.1)						Military Disputes
	Alliance By Directed	Alliance By Directed (Yearly)	Alliance By Dyad	Alliance By Dyad (Yearly)	Alliance By Member	Alliance By Member (Yearly)	Dyadic MID
Nodes	859	899	860	899	872	912	3301
No. of Edges	5814	11676	5815	11674	6271	12132	40419
Average Degree	13.537	25.976	13.523	25.971	14.383	26.605	24.489

Average Weighted Degree	232.792	5422.578	123.779	2711.362	56.612	945.969	161.853
Network Diameter	4	4	4	4	4	4	4
Density	0.016	0.029	0.016	0.029	0.017	0.029	0.007
Average Clustering Coefficient	0.162	0.149	0.162	0.148	0.158	0.151	0.237
Average Path length	2.827	2.465		2.4668	2.822	2.4656	2.5168
Number of Weakly Connected Components	2	2	2	2	2	2	2
Components	0.979045	0.978865	0.977907	0.978865	0.973624	0.973684	0.983338
No. of triangles	33.48894	65.37486	33.45	65.03115	37.8945	69.33882	94.5468

Table 2. Different Centrality Metrics

Centrality Metrics	AB Directed	AB Directed (Yearly)	AB Dyad	AB Dyad (Yearly)	AB Member	AB Member (Yearly)	Dyadic MID
Hubs	0.02106	0.022522	0.021036	0.022493	0.020875	0.02242	0.011341
Authority	0.02106	0.022522	0.021036	0.022493	0.020875	0.02242	0.011341
PageRank		0.001099	0.001155	0.00063	0.001147	0.001096	0.000303
Betweenness Centrality	751.596	631.2684	750.7419	631.3204	752.6411	633.3289	2420.817
Closeness Centrality	0.365394	0.417663	0.366288	0.418087	0.367481	0.418632	0.408386
Eigenvector Centrality	0.0067	0.072524	0.0068 0.060724	0.072661	0.062726	0.073134	0.040521

Eccentricity	3.464494	3.270601	3.462791	3.459399	3.458716	3.265351	3.269312
Harmonic closeness centrality	0.384718	0.439509	0.385567	0.439981	0.387019	0.440571	0.422883

Table 3. Number of Communities and Modularity Metrics

	AB Directed	AB Directed (Yearly)	AB Dyad	By Dyad (Yearly)	AB Member	AB Member (Yearly)	Dyadic MID
Number of Countries	859	899	860	899	872	912	3301
Modularity	0.279	0.252	0.258	0.251	0.187	0.236	0.143
Number of Communities	7	7	7	6	11	8	11

Table 4. An Example of General Characteristics of an Elementary School Friendship network, its centralities and modularity.

Network Characteristics	Elementary School (E1)	
Nodes	108	
No. of Arcs	1006	
Average Weighted Degree	18.62	
No. Components	1	
Density	0.087	

Clustering Coefficient	0.294	
Average Path length	2.477	
Betweenness Centrality	0.035	
Closeness Centrality	0.161	
Degree Centrality	10.887	
Modularity (VOS Quality)	0.507	
Number of Communities	8	
Modularity (Louvain Community Detection)	0.433	
Number of Communities	6	

Conclusion

From the study of IR networks presented we can conclude that, in general, the differences in topologies between the FIA networks and dyad MID networks are an effect attributable to the concept of IR, which changes over a lifetime. However, a bigger effect is visible when communities in the networks are studied as the basis of alliance types and disputes; it is concluded, on the basis of the network analysis, the concept of IR is clearly different at both alliance and dispute levels. This difference in the country size and composition is interesting with respect to the behavior of the countries evaluated when observed from the IR concept evaluation perspective.

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